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## SOLUTIONS OF PROBLEMS.

2689 [April, 1918]. Proposed by E. V. HUNTINGTON, Harvard University.

Show that the maximum value of

$$y = \frac{\sin x \cos (x + \varphi)}{\cos (x + \theta) \sin (x + \varphi + \theta)}$$

is  $y_1 = (\cos \theta - \rho)/(\cos \theta + \rho)$  where  $\rho = \sqrt{\sin^2 \varphi - \sin^2 \theta}$ .

This problem was suggested to the proposer by a professor of civil engineering, and has important applications in the theory of conjugate stresses.

SOLUTION BY J. B. REYNOLDS, Lehigh University.

$$y = \frac{\sin x \cos (x + \varphi)}{\cos (x + \theta) \sin (x + \varphi + \theta)}. \quad (1)$$

Taking the logarithm of (1), differentiating and setting the result equal to zero we get

$$\cot x - \tan (x + \varphi) + \tan (x + \theta) - \cot (x + \varphi + \theta) = 0 \quad (2)$$

or

$$\cot x - \cot (x + \varphi + \theta) = -\tan (x + \theta) + \tan (x + \varphi);$$

whence

$$\frac{\sin (\varphi + \theta)}{\sin (\varphi - \theta)} = \frac{\sin x \sin (x + \varphi + \theta)}{\cos (x + \varphi) \cos (x + \theta)}. \quad (3)$$

Or writing (2) in the form

$$\cot x + \tan (x + \theta) = \tan (x + \varphi) + \cot (x + \varphi + \theta)$$

we get

$$\frac{\cos \theta}{\sin x \cos (x + \theta)} = \frac{\cos \theta}{\cos (x + \varphi) \sin (x + \varphi + \theta)};$$

or

$$\frac{\cos (x + \varphi)}{\cos (x + \theta)} = \frac{\sin x}{\sin (x + \varphi + \theta)}. \quad (4)$$

(4) in (3) gives

$$\frac{\cos (x + \varphi)}{\sin x} = \sqrt{\frac{\sin (\varphi - \theta)}{\sin (\varphi + \theta)}};$$

whence

$$\cot x = \tan \varphi + \sec \varphi \sqrt{\frac{\sin (\varphi - \theta)}{\sin (\varphi + \theta)}}. \quad (5)$$

By (4) and (1), we may write

$$y_1 = \frac{\sin^2 x}{\sin^2 (x + \varphi + \theta)}$$

so that by (5)

$$\frac{1}{\sqrt{y_1}} = \cos (\varphi + \theta) + \tan \varphi \sin (\varphi + \theta) + \sec \varphi \sqrt{\sin (\varphi + \theta) \sin (\varphi - \theta)} = \frac{\cos \theta + \rho}{\cos \varphi}.$$

$$\therefore y_1 = \frac{\cos^2 \varphi}{(\cos \theta + \rho)^2} = \frac{\cos^2 \theta - \rho^2}{(\cos \theta + \rho)^2} \equiv \frac{\cos \theta - \rho}{\cos \theta + \rho}.$$

To prove that we have a maximum we find:

$$\begin{aligned} \frac{dy}{dx} &= \frac{\cos \theta}{\cos (x + \theta) \sin (x + \varphi + \theta)} \left\{ \frac{\cos (x + \varphi)}{\cos (x + \theta)} - \frac{\sin x}{\sin (x + \varphi + \theta)} \right\} \\ &= \frac{\cos \theta}{\cos^2 (x + \theta) \sin^2 (x + \varphi + \theta)} \{ \sin \varphi \cos (2x + \varphi + \theta) + \sin \theta \}, \end{aligned}$$

which shows that we have either a maximum or a minimum when the brace equals zero, since it is the only variable factor of odd power. Choosing the least value of  $x$  that will make the brace zero we have that  $\cos (2x + \varphi + \theta)$  decreases as  $x$  increases so that the sign of  $dy/dx$  changes from positive to negative as  $x$  passes through this critical value.